Algebraic version of triangle inequality for side lengths of Median Triangle.

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If x, y and z are positive real numbers, prove that

$$\sqrt{4x^2+4x(y+z)+(y-z)^2} < \sqrt{4y^2+4y(z+x)+(z-x)^2} + \sqrt{4z^2+4z(x+xy)+(x-y)^2}.$$

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Let $a := y+z, b := z+x, c := x+y$. Then numbers a, b, c satisfies triangle inequalities and, therefore, can be considered as side lengths of a triangle ABC, namely $BC = a$, $CA = b, BC + a$. Let m_a, m_b, m_c be lengths of medians in $\triangle ABc$.

Note that $4x^2+4x(y+z)+(y-z)^2 = (2x+y+z)^2-4yz = (b+c)^2-\left(a^2-(b-c)^2\right) = 2(b^2+c^2)-a^2 = 4m_a^2$ and, cyclic, $4y^2+4y(z+x)+(z-x)^2 = 4m_b^2$, $4z^2+4z(x+xy)+(x-y)^2 = 4m_c^2$. Then inequality of the problem becomes $\sqrt{4m_a^2} < \sqrt{4m_b^2} + \sqrt{4m_c^2} \iff m_a < m_b + m_c$, where latter inequality holds since lengths of median satisfies triangle inequalities (see picture below).